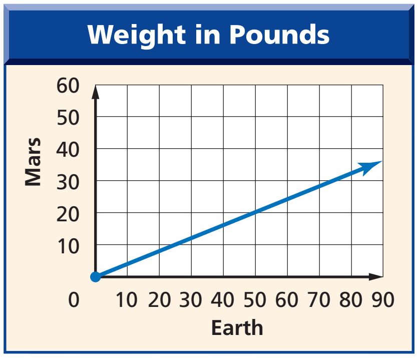
8-5 Classes of Functions

GET READY for the Lesson

Main Ideas

- Identify graphs as different types of functions.
- Identify equations as different types of functions.

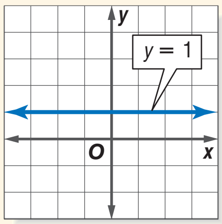
The purpose of the Mars Exploration Program is to study conditions on Mars. The findings will help NASA prepare for a possible mission with human explorers. The graph at the right compares a person's weight on Earth with his or her weight on Mars. This graph represents a direct variation, which you studied in the previous lesson.



Identify Graphs In this book, you have studied several types of graphs representing special functions. The following is a summary of these graphs.

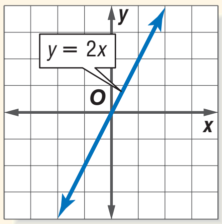
CONCEPT SUMMARY — *Special Functions*

Constant Function



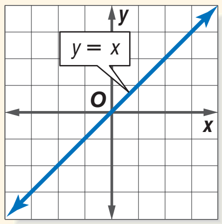
The general equation of a constant function is $y = a$, where a is any number. Its graph is a horizontal line that crosses the y-axis at a.

Direct Variation Function



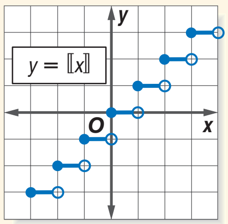
The general equation of a direct variation function is $y = ax$, where a is a nonzero constant. Its graph is a line that passes through the origin and is neither horizontal nor vertical.

Identity Function



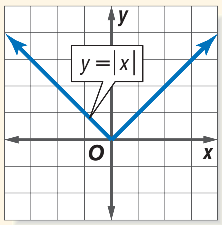
The identity function $y = x$ is a special case of the direct variation function in which the constant is 1. Its graph passes through all points with coordinates (a, a).

Greatest Integer Function



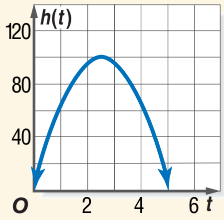
If an equation includes an expression inside the greatest integer symbol, the function is a greatest integer function. Its graph looks like steps.

Absolute Value Function



An equation with the independent variable inside absolute value symbols is an absolute value function. Its graph is in the shape of a V.

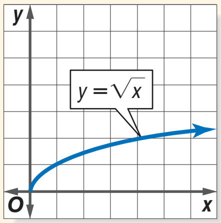
Quadratic Function



The general equation of a quadratic function is $y = ax^2 + bx + c$, where $a \neq 0$. Its graph is a parabola.

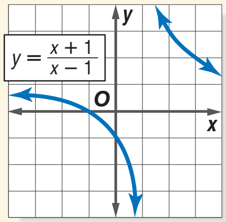
CONCEPT SUMMARY — Special Functions

Square Root Function



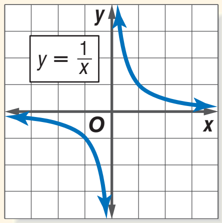
If an equation includes the independent variable inside the radical sign, the function is a square root function. Its graph is a curve that starts at a point and continues in only one direction.

Rational Function



The general equation for a rational function is $y = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions. Its graph may have one or more asymptotes and/or holes.

Inverse Variation Function



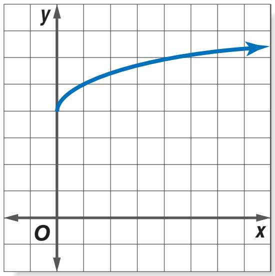
The inverse variation function $y = \frac{a}{x}$ is a special case of the rational function where $p(x)$ is a constant and $q(x) = x$. Its graph has two asymptotes, $x = 0$ and $y = 0$.

You can use the shape of the graphs of each type of function to identify the type of function that is represented by a given graph. To do so, keep in mind the graph of the parent function of each function type.

EXAMPLE 1 Identify a Function Given the Graph

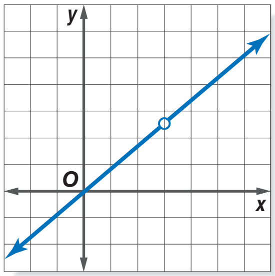
Identify the type of function represented by each graph.

a.



The graph has a starting point and curves in one direction. The graph represents a square root function.

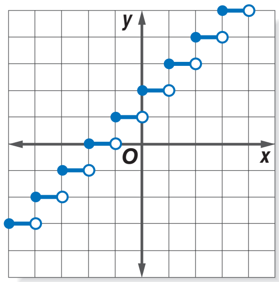
b.



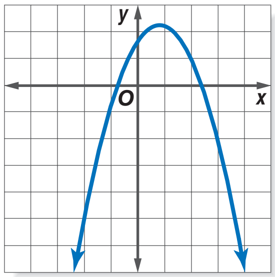
The graph appears to be a direct variation since it is a straight line passing through the origin. However, the hole indicates that it represents a rational function.

CHECK Your Progress

1A.



1B.



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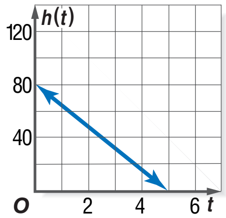
Identify Equations If you can identify an equation as a type of function, you can determine the shape of the graph.

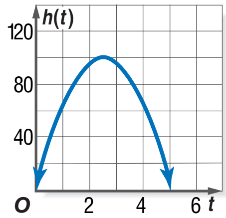
EXAMPLE Match Equation with Graph

2 **ROCKETRY** **Emily launched a toy rocket from ground level. The height above the ground level h, in feet, after t seconds is given by the formula $h(t) = -16t^2 + 80t$. Which graph depicts the height of the rocket during its flight?**

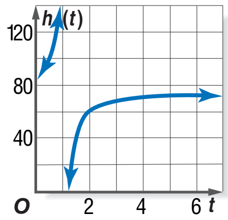
a.



b.



c.



The function includes a second-degree polynomial. Therefore, it is a quadratic function, and its graph is a parabola. Graph **b** is on the only parabola. Therefore, the answer is graph **b**.

CHECK Your Progress

2. Which graph above could represent an elevator moving from a height of 80 feet to ground level in 5 seconds?

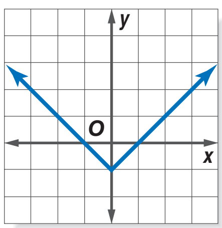
Sometimes recognizing an equation as a specific type of function can help you graph the function.

EXAMPLE Identify a Function Given its Equation

3 **Identify the type of function represented by each equation. Then graph the equation.**

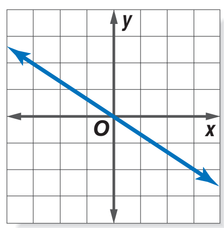
a. $y = |x| - 1$

Since the equation includes an expression inside absolute value symbols, it is an absolute value function. Therefore, the graph will be in the shape of a V. Plot some points and graph the absolute value function.



b. $y = -\frac{2}{3}x$

The function is in the form $y = ax$, where $a = -\frac{2}{3}$. Therefore, it is a direct variation function. The graph passes through the origin and has a slope of $-\frac{2}{3}$.



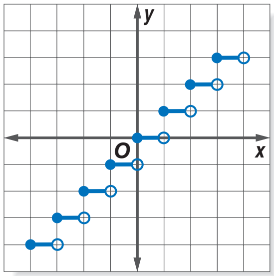
CHECK Your Progress

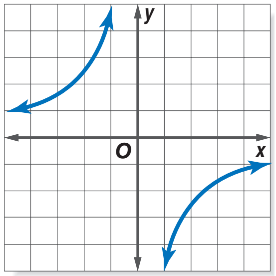
3A. $y = [\![x - 1]\!]$

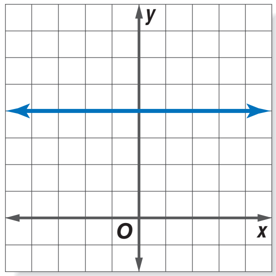
3B. $y = \frac{-1}{x + 1}$

CHECK Your Understanding

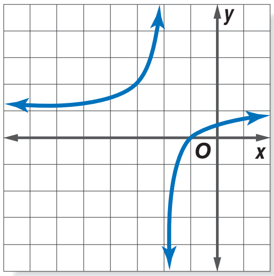
Example 1 (p. 474) **Identify the type of function represented by each graph.**

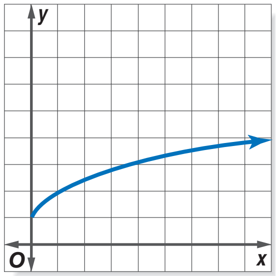
1. 

2. 

3. 

Example 2 (p. 475) **Match each graph with an equation at the right.**

4. 

5. 

a. $y = x^2 + 2x + 3$
b. $y = \sqrt{x} + 1$
c. $y = \dfrac{x+1}{x+2}$
d. $y = [\![2x]\!]$

6. **GEOMETRY** Write the equation for the area of a circle. Identify the equation as a type of function. Describe the graph of the function.

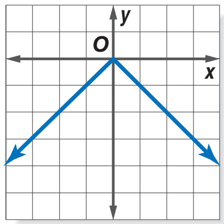
Example 3 (p. 475) **Identify the type of function represented by each equation. Then graph the equation.**

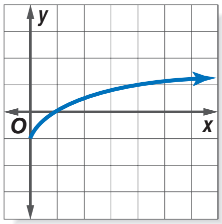
7. $y = x$
8. $y = -x^2 + 2$
9. $y = |x + 2|$

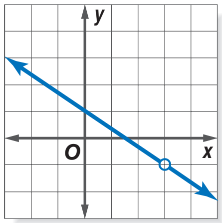
Exercises

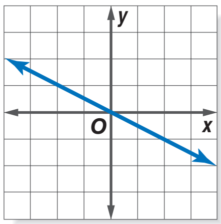
HOMEWORK HELP	
For Exercises	See Examples
10–15	1
16–23	3
24–31	2

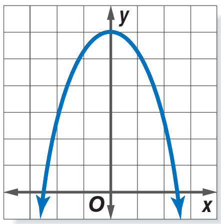
Identify the function represented by each graph.

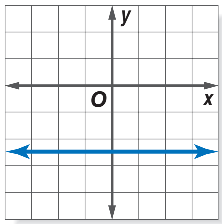
10. 

11. 

12. 

13. 

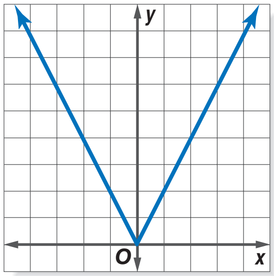
14. 

15. 

Identify the type of function represented by each equation. Then graph the equation.

16. $y = -1.5$
17. $y = 2.5x$
18. $y = \sqrt{9x}$
19. $y = \frac{4}{x}$
20. $y = \dfrac{x^2 - 1}{x - 1}$
21. $y = 3[\![x]\!]$
22. $y = |2x|$
23. $y = 2x^2$

Match each graph with an equation at the right.

24.


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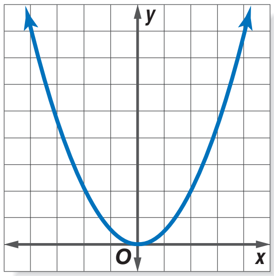
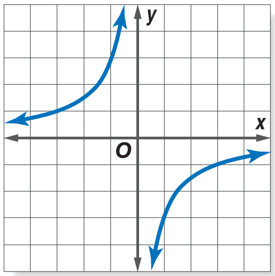
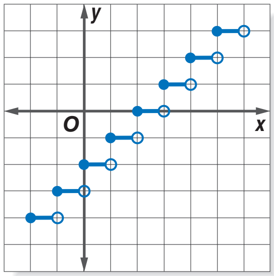


26.


27.


a. $y = [\![x]\!] - 2$
b. $y = 2|x|$
c. $y = 2\sqrt{x}$
d. $y = -3x$
e. $y = 0.5x^2$
f. $y = -\frac{3}{x+1}$
g. $y = -\frac{3}{x}$

Real-World Link
When the Hope Diamond was shipped from New York to the Smithsonian Institution in Washington, D.C., it was mailed in a plain brown paper package.

Source: usps.com

HEALTH **For Exercises 28–30, use the following information.**
A woman painting a room will burn an average of 4.5 Calories per minute.

28. Write an equation for the number of Calories burned in m minutes.

29. Identify the equation in Exercise 28 as a type of function.

30. Describe the graph of the function.

31. **ARCHITECTURE** The shape of the Gateway Arch of the Jefferson National Expansion Memorial in St. Louis, Missouri, resembles the graph of the function $f(x) = -0.00635x^2 + 4.0005x - 0.07875$, where x is in feet. Describe the shape of the Gateway Arch.

MAIL **For Exercises 32 and 33, use the following information.**
In 2006, the cost to mail a first-class letter was 39¢ for any weight up to and including 1 ounce. Each additional ounce or part of an ounce added 24¢ to the cost.

32. Make a graph showing the postal rates to mail any letter from 0 to 8 ounces.

33. Compare your graph in Exercise 32 to the graph of the greatest integer function.

EXTRA PRACTICE
See pages 909, 933.
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H.O.T. Problems

34. **OPEN ENDED** Find a counterexample to the statement *All functions are continuous*. Describe your function.

35. **CHALLENGE** Identify each table of values as a type of function.

a.

x	f(x)
−5	7
−3	5
−1	3
0	2
1	3
3	5
5	7
7	9

b.

x	f(x)
−5	24
−3	8
−1	0
0	−1
1	0
3	8
5	24
7	48

c.

x	f(x)
−1.3	−1
−1.7	−1
0	1
0.8	1
0.9	1
1	2
1.5	2
2.3	3

d.

x	f(x)
−5	undefined
−3	undefined
−1	undefined
0	0
1	1
4	2
9	3
16	4

36. **CHALLENGE** Without graphing either function, explain how the graph of $y = [\![x + 2]\!] - 3$ is related to the graph of $y = [\![x + 1]\!] - 1$.

37. ***Writing in Math*** Use the information on page 473 to explain how the graph of a function can be used to determine the type of relationship that exists between the quantities represented by the domain and the range.

STANDARDIZED TEST PRACTICE

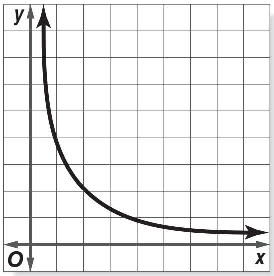
38. **ACT/SAT** The curve below could be part of the graph of which function?

A $y = \sqrt{x}$

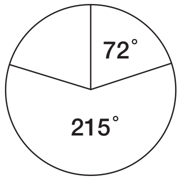
B $y = x^2 - 5x + 4$

C $xy = 4$

D $y = -x + 20$



39. **REVIEW** A paper plate with a 12-inch diameter is divided into 3 sections.



What is the approximate length of the arc of the largest section?

F 20.3 inches

G 22.5 inches

H 24.2 inches

J 26.5 inches

Spiral Review

40. If x varies directly as y and $y = \frac{1}{5}$ when $x = 11$, find x when $y = \frac{2}{5}$. (Lesson 8-4)

Graph each rational function. (Lesson 8-3)

41. $f(x) = \frac{3}{x + 2}$

42. $f(x) = \frac{8}{(x - 1)(x + 3)}$

43. $f(x) = \frac{x^2 - 5x + 4}{x - 4}$

Solve each equation by factoring. (Lesson 5-2)

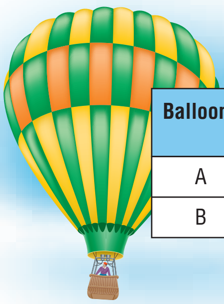
44. $x^2 + 6x + 8 = 0$

45. $2q^2 + 11q = 21$

HOT-AIR BALLOONS For Exercises 46 and 47, use the table. (Lesson 3-2)

46. If both balloons are launched at the same time, how long will it take for them to be the same distance from the ground?

47. What is the distance of the balloons from the ground at that time?



Balloons	Distance from Ground (m)	Rate of Ascension (m/min)
A	60	15
B	40	20

GET READY for the Next Lesson

PREREQUISITE SKILL Find the LCM of each set of polynomials. (Lesson 8-2)

48. $15ab^2c, 6a^3, 4bc^2$

49. $9x^3, 5xy^2, 15x^2y^3$

50. $5d - 10, 3d - 6$

51. $x^2 - y^2, 3x + 3y$

52. $a^2 - 2a - 3, a^2 - a - 6$

53. $2t^2 - 9t - 5, t^2 + t - 30$